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Today

- Image Restoration

Image Enhancement

- Image enhancement techniques basically are heuristic procedures designed to manipulate an image in order to take advantage of the psychophysical aspects of the human visual system
- Image enhancement is largely a subjective process
- Stretching, denoising, smoothing, sharpening, …

Image Restoration

- Image restoration attempts to reconstruct or recover an image that has been degraded by using a priori knowledge of the degradation phenomenon
- Image restoration is for the most part an objective process
- Goal of image restoration
  - to improve the quality of an image

Image Restoration

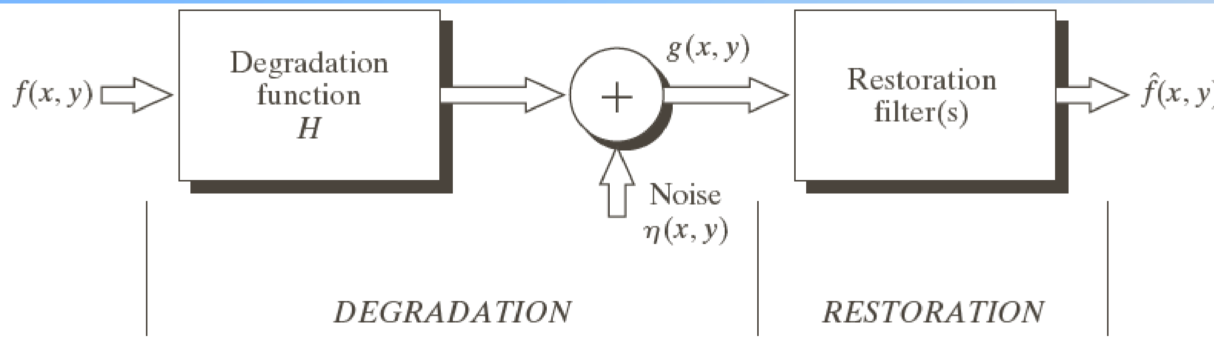
- Image restoration techniques are oriented toward modeling the degradation and applying the inverse process in order to recover the original image

Image Restoration

- The purpose of image restoration is to "compensate for" or "undo" defects which degrade an image. Degradation comes in many forms such as motion blur, noise, and camera misfocus. In cases like motion blur, it is possible to come up with an very good estimate of the actual blurring function and "undo" the blur to restore the original image. In cases where the image is corrupted by noise, the best we may hope to do is to compensate for the degradation it caused

Model of the Image Degradation/Restoration Process

FIGURE 5.1 A model of the image degradation/restoration process.



The degradation process is modeled as a degradation function that, together with an additive noise term, operates on an input image *f(x, y)* to produce a degraded image *g(x, y)*

The objective of image restoration is to obtain an estimate $\hat{f}(x, y)$ of the original input image

Model of the Image Degradation/Restoration Process

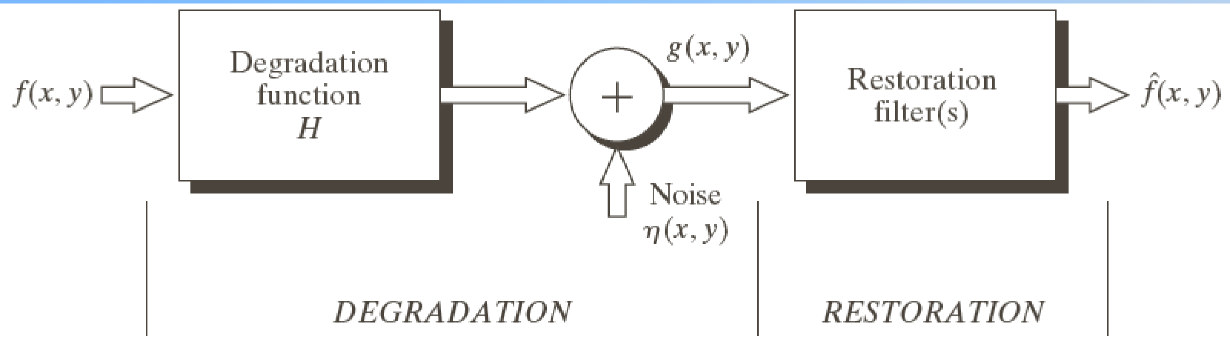


FIGURE 5.1 A model of the image degradation/restoration process.

- If H is a linear, position-invariant process, then the degraded image is given in the spatial domain by

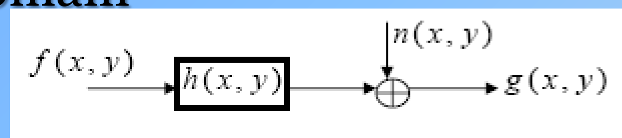
$$g(x,y)=h(x,y)*f(x,y)+\eta(x,y)$$

- In the frequency domain

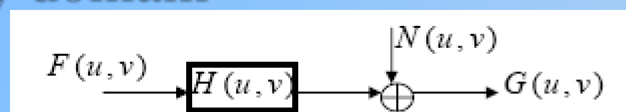
$$G(x,y)=H(x,y)F(x,y)+N(x,y)$$

Model of the Image Degradation/Restoration Process

- In the spatial domain



- In the frequency domain



Our purpose is to recover $f(x, y)$ from the noise image $g(x, y)$, which is almost the same as to remove noise $\eta(x, y)$ from $g(x,y)$ if we don't consider the impact of $h(x, y)$.

To remove noise efficiently, it is better to know the noise model first;

To build a model for an unknown noise image, it is better to know all the existing and widely used noise models.

Noise Models

- Some important noise probability density functions

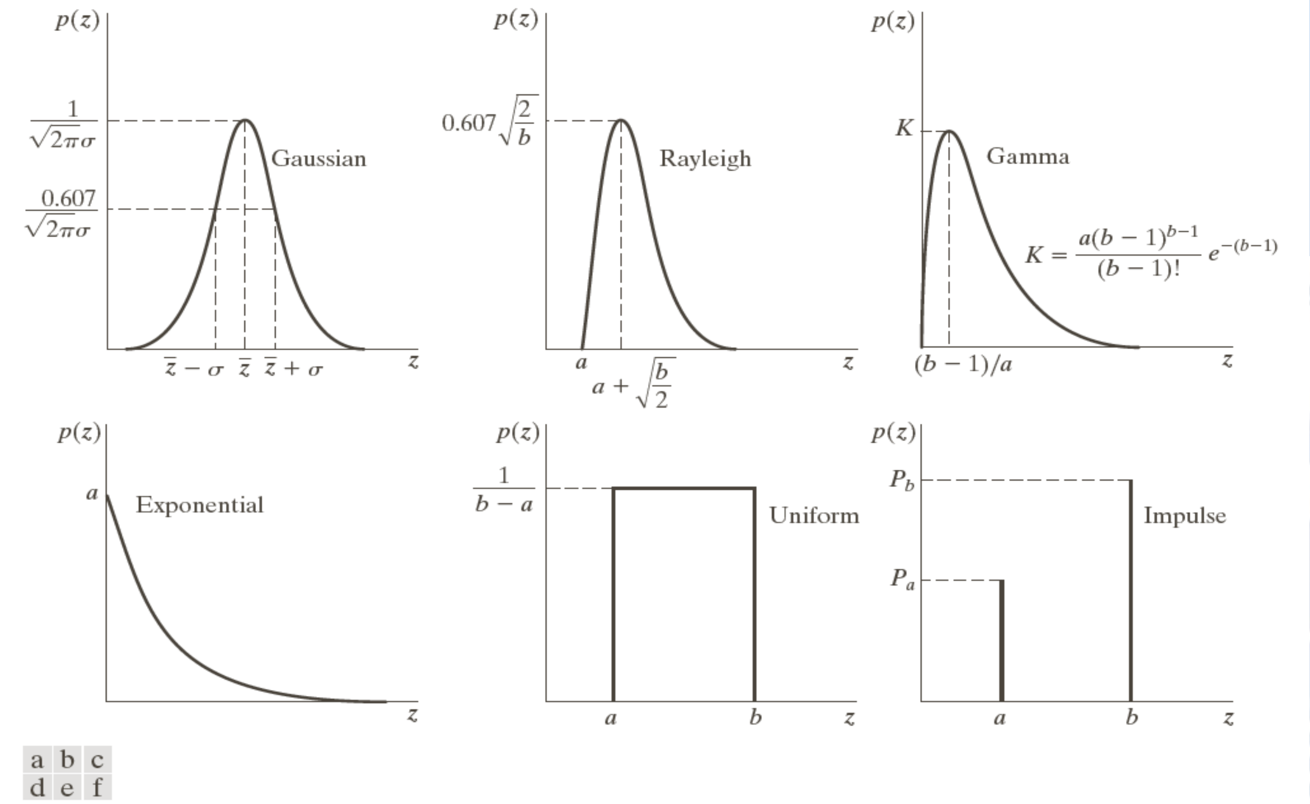


FIGURE 5.2 Some important probability density functions.

Noise Models

- Gaussian noise

$$p(z)=\frac{1}{\sqrt{2\pi\sigma}}e^{-(z-\bar{z})^2/2\sigma^2}$$

Noise Models

- Rayleigh noise

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \ge a \\ 0 & \text{for } z < a \end{cases}$$

- The mean and variance of this density is given by

$$\bar{z} = a + \sqrt{\pi b/4}$$

- and

$$\sigma^2 = \frac{b(4-\pi)}{4}$$

Noise Models

- Erlang(gamma) noise

$$p(z)=\begin{cases}\dfrac{a^b z^{b-1}}{(b-1)!}e^{-az} & \text{for } z\geq 0\\ 0 & \text{for } z<0\end{cases}$$

- The mean and variance of this density is given by

$$\bar{z}=\frac{b}{a}$$

- and

$$\sigma^2=\frac{b}{a^2}$$

Noise Models

- Exponential noise

$$p(z)=\begin{cases} ae^{-az} & \text{for } z \ge 0 \\ 0 & \text{for } z < 0 \end{cases}$$

- The mean and variance of this density is given by

$$\bar{z}=\frac{1}{a}$$

- and

$$\sigma^2=\frac{1}{a^2}$$

Noise Models

- Uniform noise

$$p(z)=\begin{cases}\dfrac{1}{b-a} & if\ \ a\le z\le b\\ 0 & otherwise\end{cases}$$

- The mean and variance of this density is given by

$$\bar{z}=\frac{a+b}{2}$$

- and

$$\sigma^2=\frac{(b-a)^2}{12}$$

Noise Models

- Impulse (salt-and-pepper) noise

$$p(z)=\begin{cases} P_a & \text{for } z=a \\ P_b & \text{for } z=b \\ 0 & \text{otherwise} \end{cases}$$

- The salt-and-pepper appearance of the image corrupted by impulse is the only one that is visually indicative of the type of noise causing the degradation

Noise Models

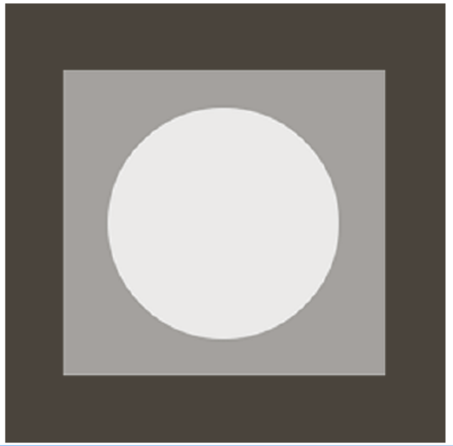


FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

noise image and their histograms

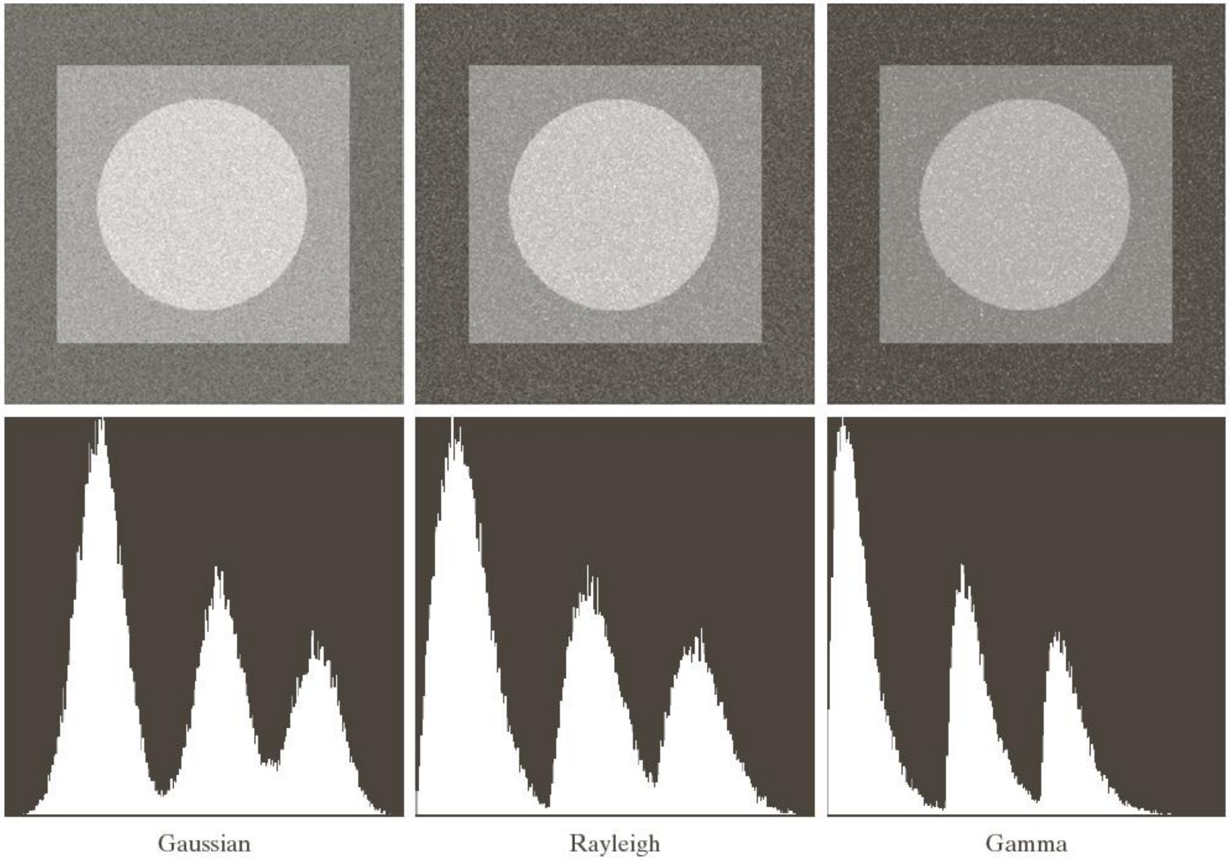


FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

noise image and their histograms

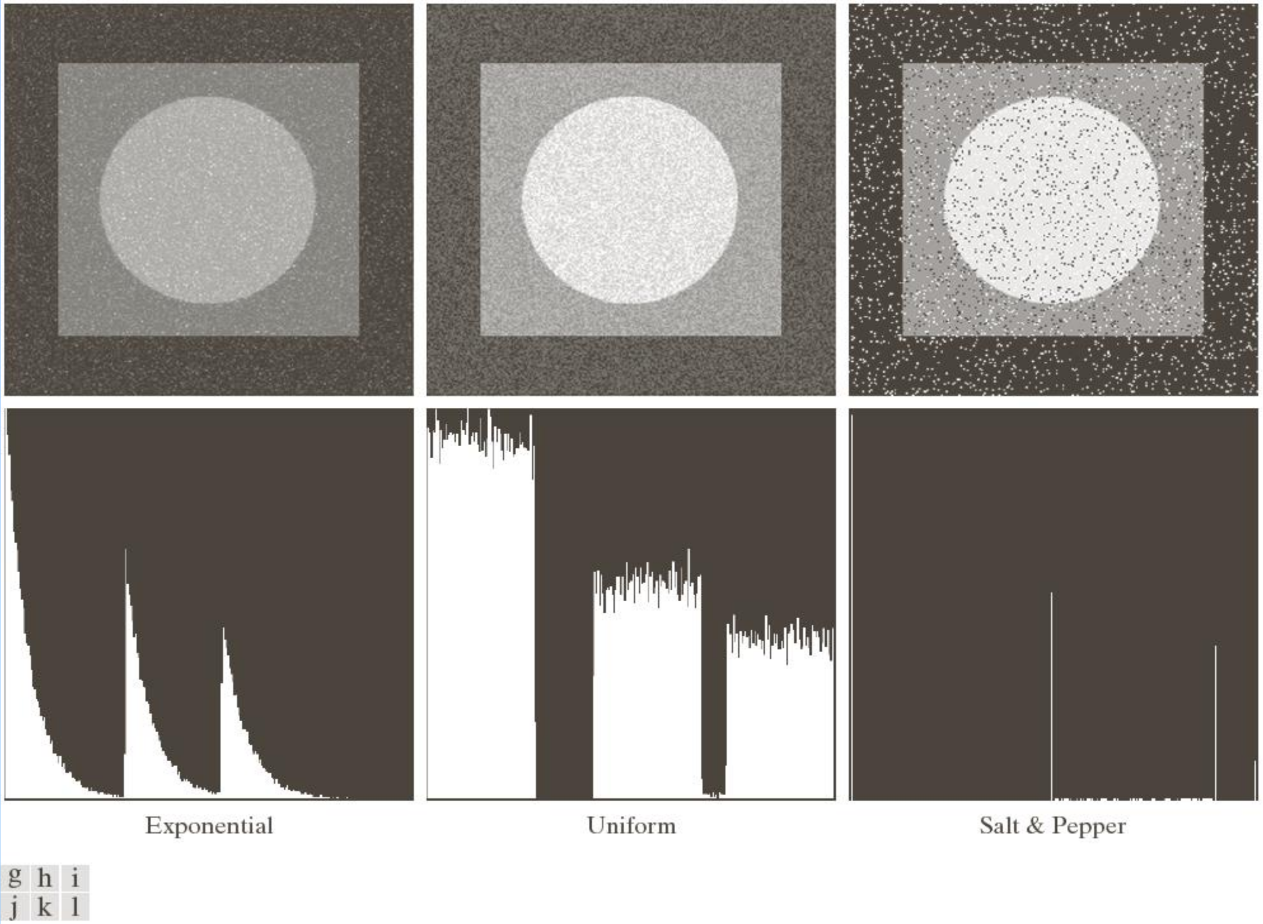
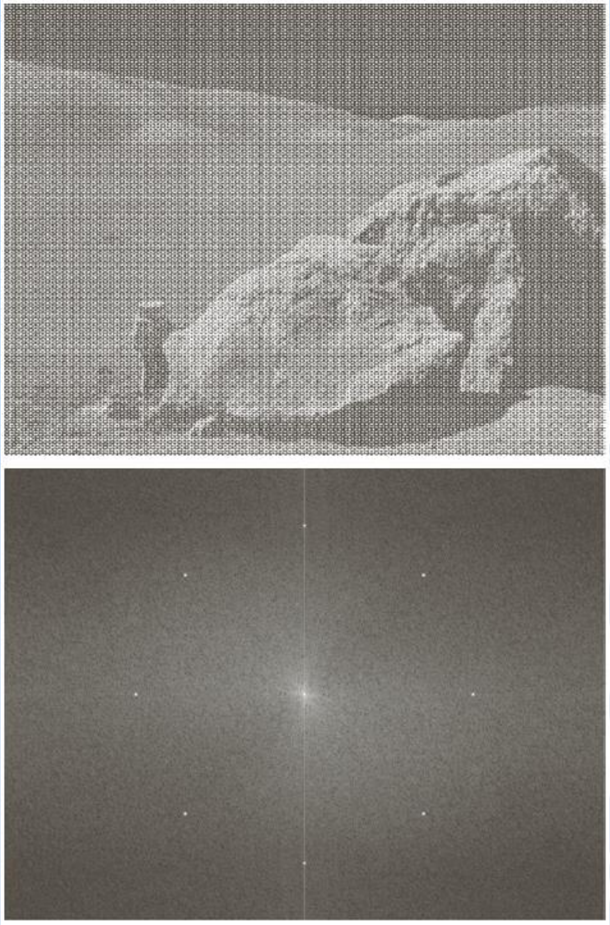


FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and salt and pepper noise to the image in Fig. 5.3.

Noise Models

- Periodic noise
  - Periodic noise in an image arises typically from electrical or eletromechanical interference during image acquisition. This is the only type of spatial dependent noise that will be considered in this chapter
  - The DFT of the original image shows four conjugate pair of peaks indicating the frequencies of the periodic noise in the original image.



a
b

FIGURE 5.5 (a) Image corrupted by sinusoidal noise. (b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)

Noise Models

- Estimation of noise parameters
  - The parameters of periodic noise typically are estimated by inspection of the Fourier spectrum of the image.
  - The simplest use of the data from the image strips is for calculating the mean and variance of intensity levels.
  - We estimate the mean and variance as follows, where S is a trip, and
  - $p_s(z_i)$ denote the probability estimates(normalized histogram values) of the intensities of the pixels in S, L is the number of possible intensities in the
  - entire image.

$$\bar{z} = \sum_{i=0}^{L-1} z_i p_s(z_i) \qquad \sigma^2 = \sum_{i=0}^{L-1} (z_i - \bar{z})^2 p_S(z_i)$$

Noise Models

- Estimation of noise parameters

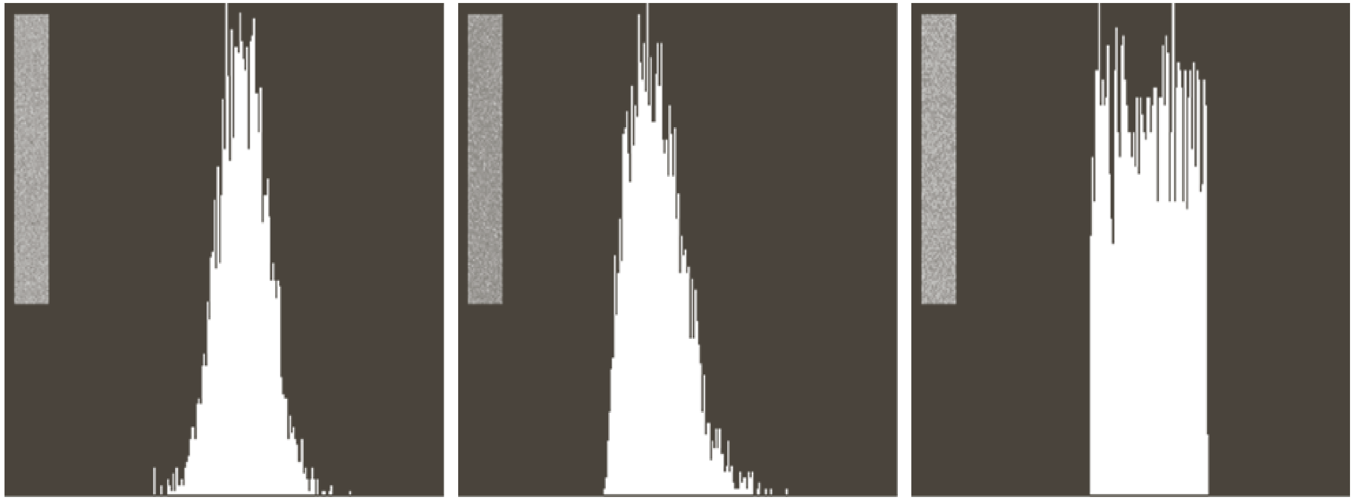


FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

Noise Models

Given the probability density function measured from the histogram of noise using the test pattern shown above, first the most-likely noise model is chosen (be it Gaussian, Rayleigh, Gamma or exponential) before the noise parameters are estimated.

Then, we can use the image strip, denoted by S, to calculate the *mean and variance* of the gray level.

$$\bar{z} = \sum_{i=0}^{L-1} z_i p_s(z_i) \qquad \sigma^2 = \sum_{i=0}^{L-1} (z_i - \bar{z})^2 p_S(z_i)$$

Restoration in the Presence of Noise Only—Spatial Filtering

- When the only degradation present in an image is noise, the degraded image is given by

$$g(x,y) = f(x,y) + \eta(x,y)$$

and

$$G(u,v) = F(u,v) + N(u,v)$$

- Spatial filtering is the method of choice in situations when only additive random noise is present, which is discussed in detail in Chapter 3

Restoration in the Presence of Noise Only—Spatial Filtering

- Mean filters
  - Arithmetic mean filter

$$\hat{f}(x,y)=\frac{1}{mn}\sum_{(s,t)\in S_{xy}} g(s,t)$$

  - Geometric mean filter

$$\hat{f}(x,y)=\left[\prod_{(s,t)\in S_{xy}} g(s,t)\right]^{\frac{1}{mn}}$$

Restoration in the Presence of Noise Only—Spatial Filtering

- Mean filters
  - Harmonic mean filter

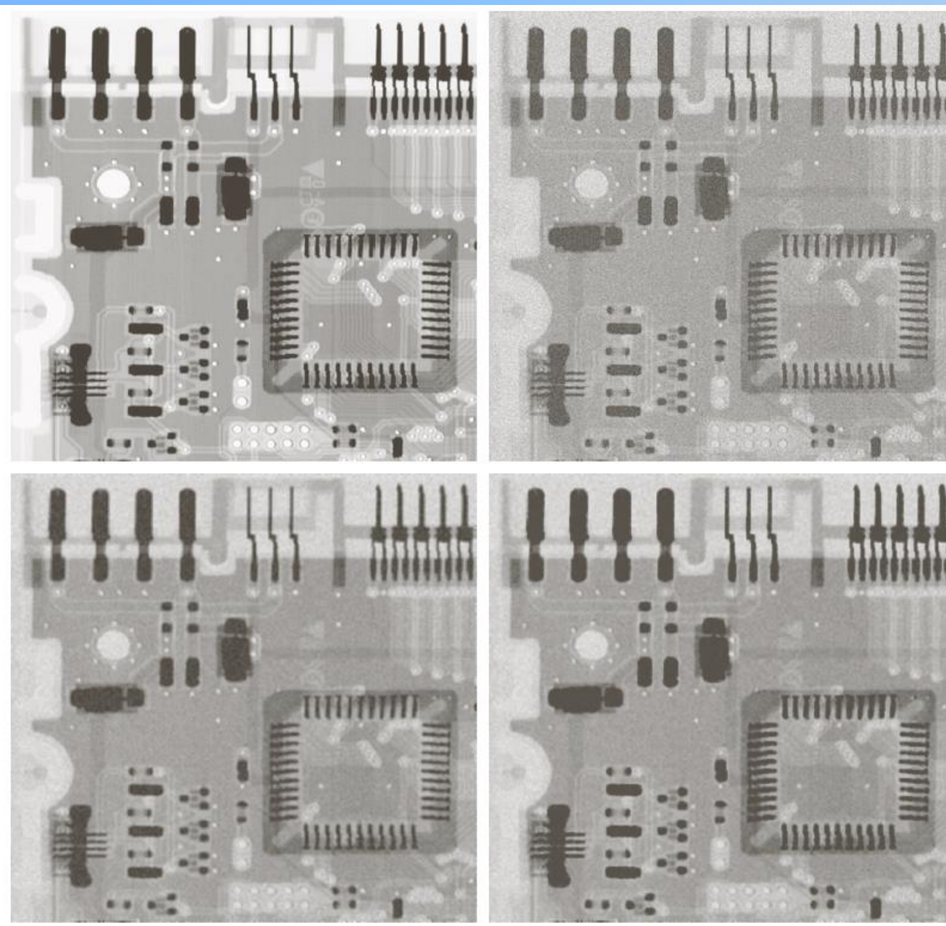
$$\hat{f}(x,y)=\frac{mn}{\sum_{(s,t)\in S_{xy}}\frac{1}{g(s,t)}}$$

  - Contra harmonic mean filter

$$\hat{f}(x,y)=\frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

- $Q>0$: for pepper noise
- $Q<0$: for salt noise
- $Q=0$: arithmetic mean filter
- $Q=-1$: harmonic mean filter

Restoration in the Presence of Noise Only—Spatial Filtering

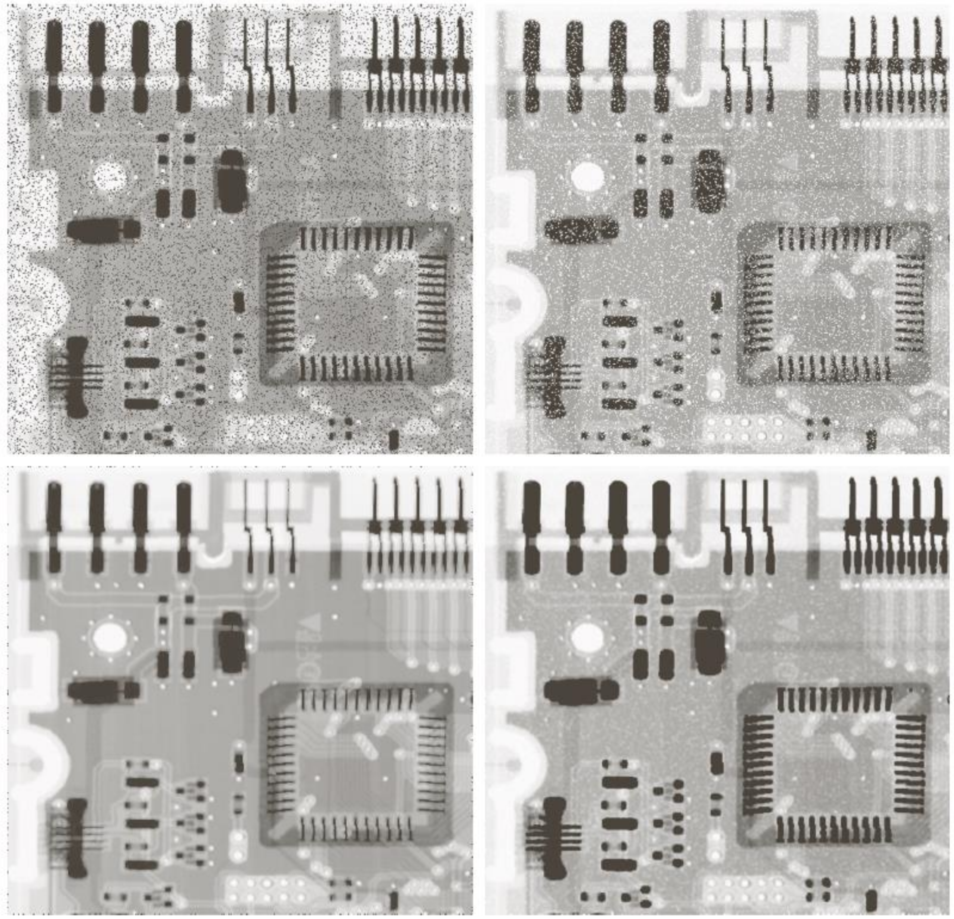


a b
c d

FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 × 3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Restoration in the Presence of Noise Only—Spatial Filtering

Correct parameters Q

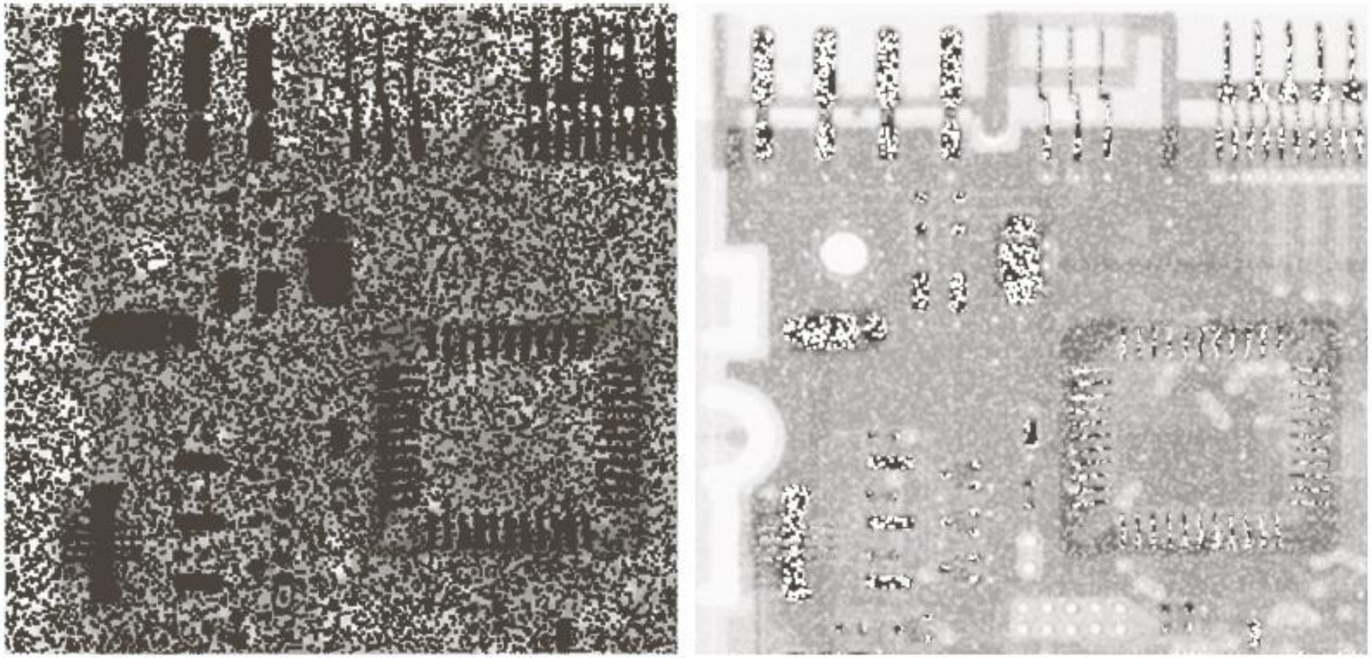


a b
c d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3 × 3 contraharmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

Restoration in the Presence of Noise Only—Spatial Filtering

Wrong parameters Q



a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.

Restoration in the Presence of Noise Only—Spatial Filtering

- Order-statistic filters
  - Median filter

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$

  - Max and min filters

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}}\{g(s,t)\}$$

and

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}}\{g(s,t)\}$$

Restoration in the Presence of Noise Only—Spatial Filtering

- Order-statistic filters
  - Midpoint filter

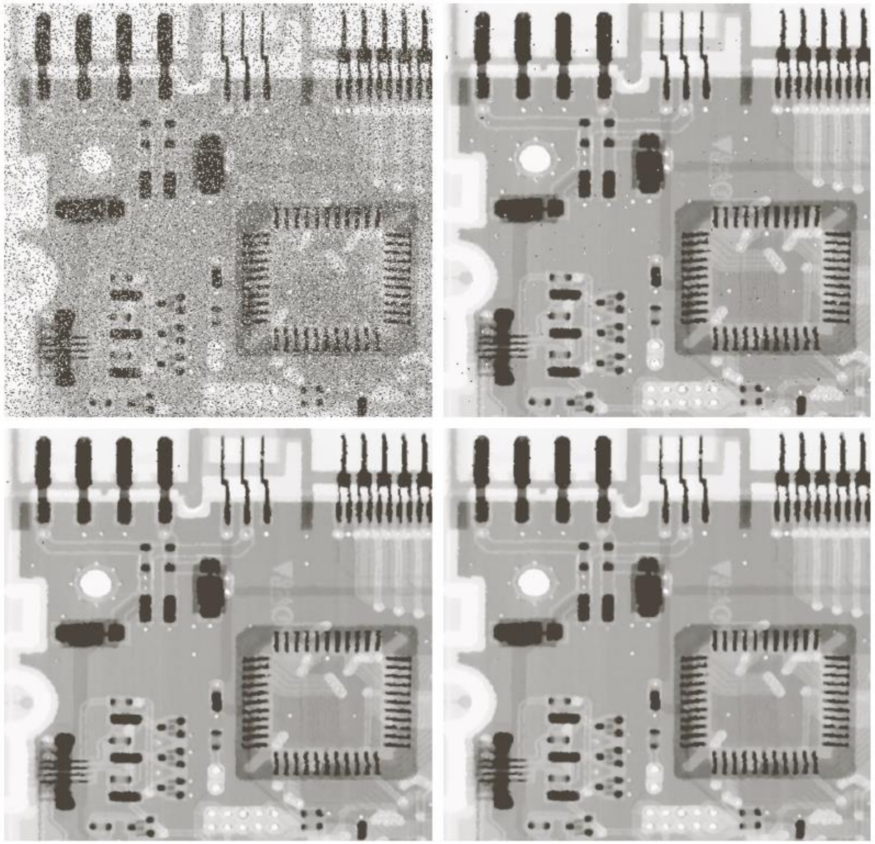
$$\hat{f}(x,y)=\frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\}+\min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

  - Alpha-trimmed mean filters

$$\hat{f}(x,y)=\frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$$

Restoration in the Presence of Noise Only—Spatial Filtering

- Order-statistic filters

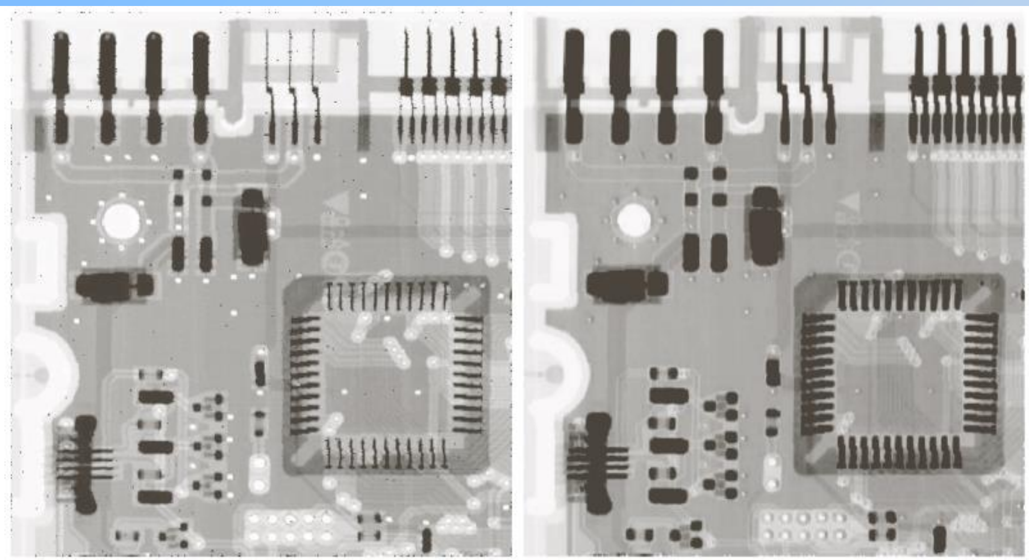


a b
c d

FIGURE 5.10 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3×3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.

Restoration in the Presence of Noise Only— Spatial Filtering

- Order-statistic filters

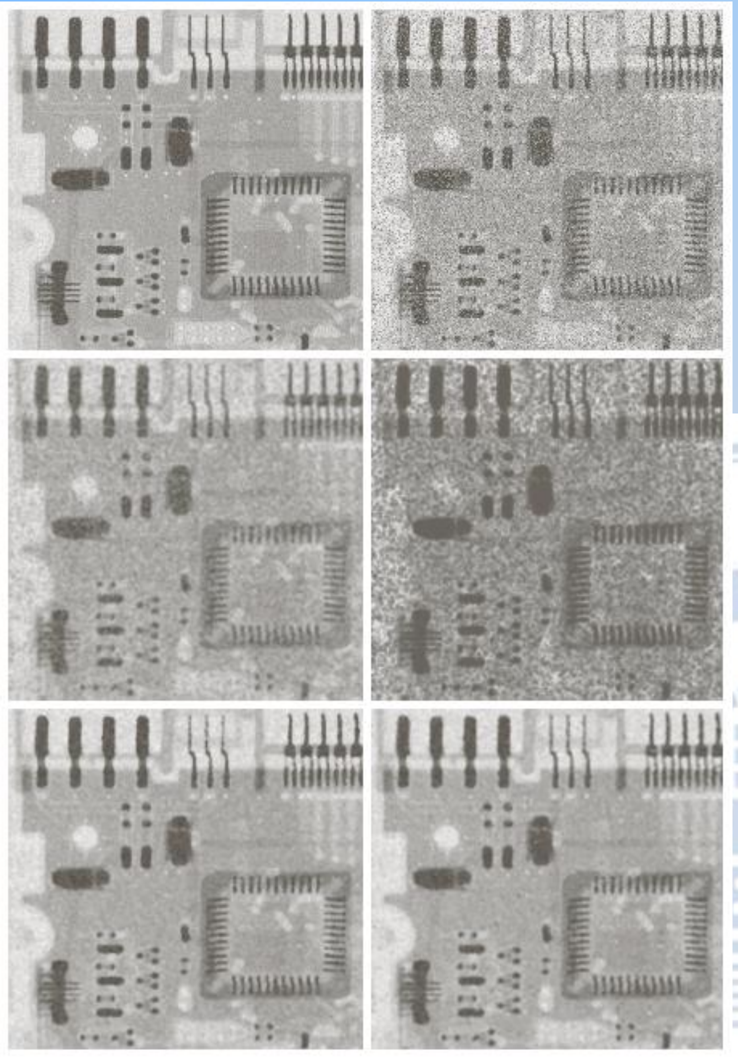


a b

FIGURE 5.11 (a) Result of filtering Fig. 5.8(a) with a max filter of size 3 × 3. (b) Result of filtering 5.8(b) with a min filter of the same size.

Restoration in the Presence of Noise Only—Spatial Filtering

- Order-statistic filters



a	b
c	d
e	f

FIGURE 5.12 (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image (b) filtered with a 5 × 5; (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.

Restoration in the Presence of Noise Only—Spatial Filtering

- Adaptive filters
  - Adaptive, local noise reduction filter

1. If σ_η^2 is zero, the filter should return simply the value of g(x, y). This is the trivial, zero-noise case in which g(x, y) is equal to f(x, y)
2. If the local variance is high relative to σ_η^2, the filter should return a value close to g(x, y). A high local variance typically is associated with edges, and these should be preserved.
3. If the two variances are equal, we want the filter to return the arithmetic mean value of the pixels in S_{xy}. This condition occurs when the local area has the same properties as the overall image, and local noise is to be reduced simply by averaging

Restoration in the Presence of Noise Only—Spatial Filtering

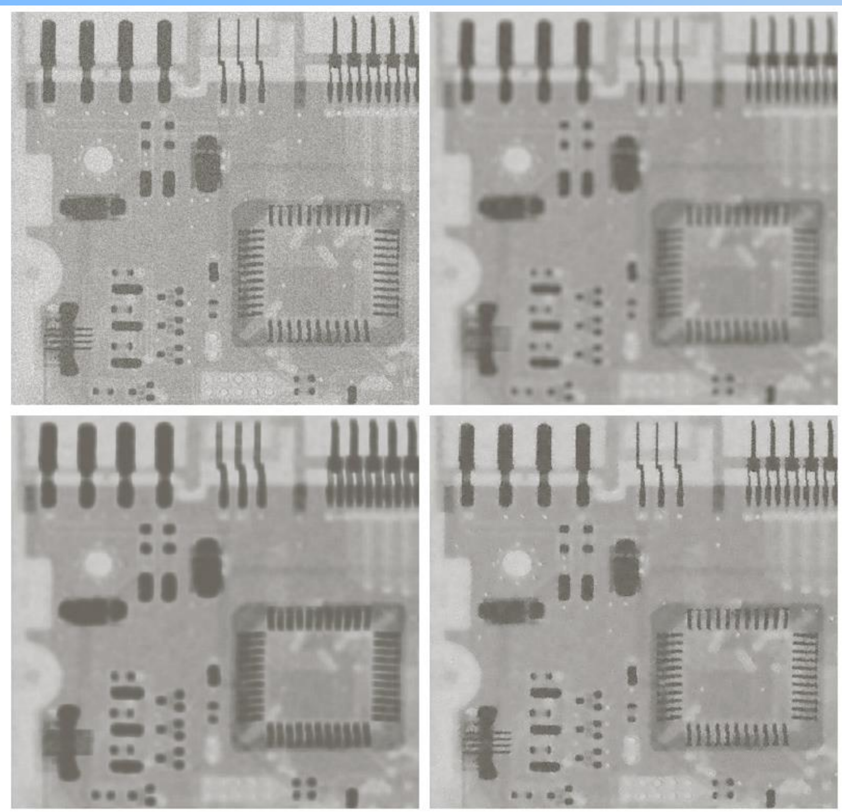
- Adaptive filters
  - Adaptive, local noise reduction filter
  - An adaptive expression based on these assumption may be written as

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x,y) - m_L\right]$$

An adaptive filter can remove Gaussian noise as effective as arithmetic and geometric mean filter, but with a minor effect of blurring (low-pass filtering).

Restoration in the Presence of Noise Only—Spatial Filtering

- Adaptive filters
  - Adaptive, local noise reduction filter



a b
c d

FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7 × 7.

Restoration in the Presence of Noise Only—Spatial Filtering

- Adaptive filters
  - Adaptive median filter
  - Stage A:

    $A1 = z_{med} - z_{min}$

    $A2 = z_{med} - z_{max}$

    If A1>0 and A2<0, go to stage B

    Else increase the window size

    If window size<= S_{max} repeat stage A

  - Stage B:

    $B1 = z_{xy} - zmin$

    $B2 = z_{xy} - zmax$

    If B1>0 and B2<0, output z_{xy}

    Else output z_{med}

Restoration in the Presence of Noise Only—Spatial Filtering

- Adaptive filters
  - Adaptive median filter

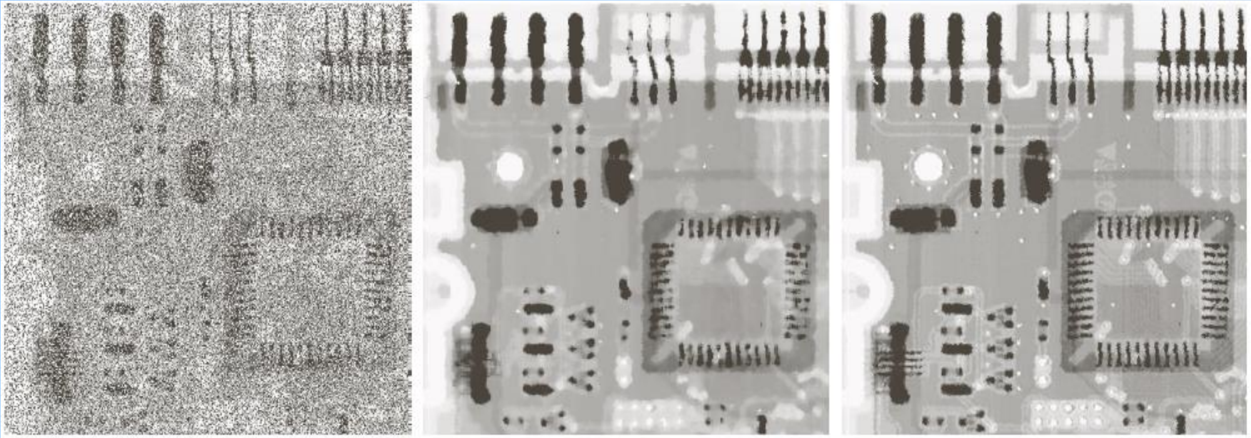


FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.

Periodic Noise Reduction by Frequency Domain Filtering

- Bandreject filters

FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.

Periodic Noise Reduction by Frequency Domain Filtering

- Bandreject filters
  - Ideal bandreject filter

$$H(u,v)=\begin{cases}1, & D(u,v) < D_0-\frac{W}{2}\\ 0, & D_0-\frac{W}{2}\le D(u,v)\le D_0+\frac{W}{2}\\ 1, & D(u,v) > D_0+\frac{W}{2}\end{cases}$$

Periodic Noise Reduction by Frequency Domain Filtering

- Bandreject filters
  - Butterworth bandreject filter

$$H(u,v)=\frac{1}{1+\left[\frac{D(u,v)W}{D^2(u,v)-D_0^2}\right]^{2n}}$$

Periodic Noise Reduction by Frequency Domain Filtering

- Bandreject filters
  - Gaussian bandreject filter

$$H(u,v)=1-e^{-\frac{1}{2}\left[\frac{D^2(u,v)-D_0^2}{D(u,v)W}\right]^2}$$

Periodic Noise Reduction by Frequency Domain Filtering

- Bandreject filters

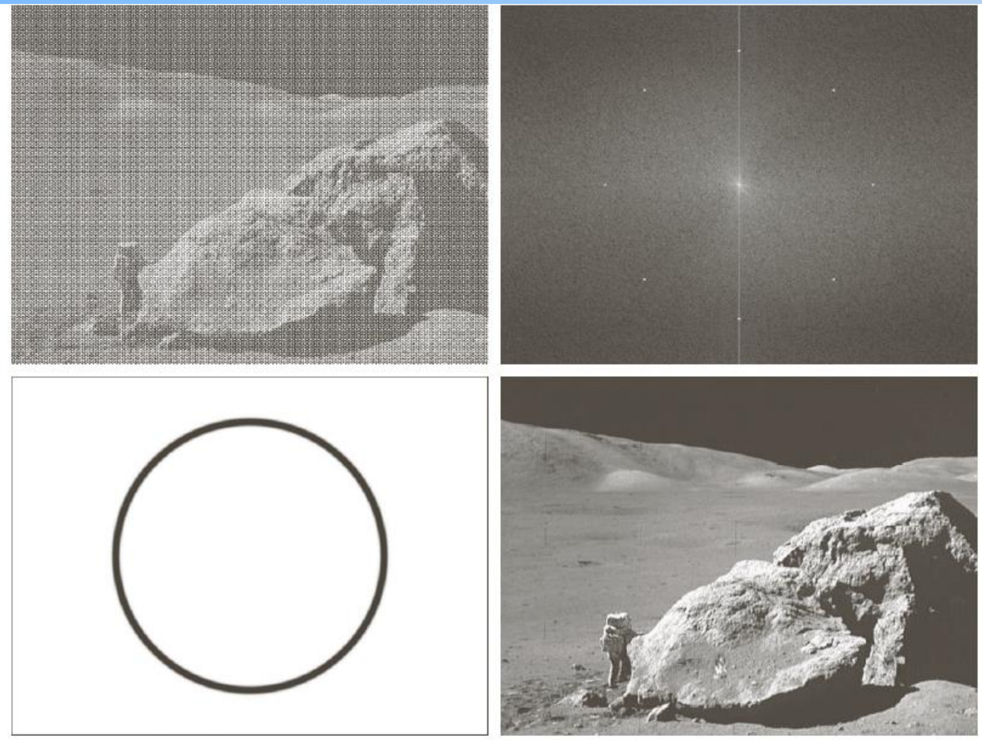


FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

Periodic Noise Reduction by Frequency Domain Filtering

- Bandpass filters
  - The transfer function $H_{BP}(u, v)$ of a bandpass filter is obtained from a corresponding bandreject filter with transfer function $H_{BR}(u, v)$ by using the equation

$$H_{BP}(u,v) = 1 - H_{BR}(u,v)$$

  - Ideal bandpass filter

$$H(u,v) = \begin{cases} 0, & D(u,v) < D_0 - \frac{W}{2} \\ 1, & D_0 - \frac{W}{2} \le D(u,v) \le D_0 + \frac{W}{2} \\ 0, & D(u,v) > D_0 + \frac{W}{2} \end{cases}$$

Periodic Noise Reduction by Frequency Domain Filtering

- Bandpass filters
  - Butterworth bandpass filter

$$H(u,v)=1-\frac{1}{1+\left[\frac{D(u,v)W}{D^2(u,v)-D_0^2}\right]^{2n}}$$

  - Gaussian bandpass filter

$$H(u,v)=e^{-\frac{1}{2}\left[\frac{D^2(u,v)-D_0^2}{D(u,v)W}\right]^2}$$

Periodic Noise Reduction by Frequency Domain Filtering

- Bandpass filters

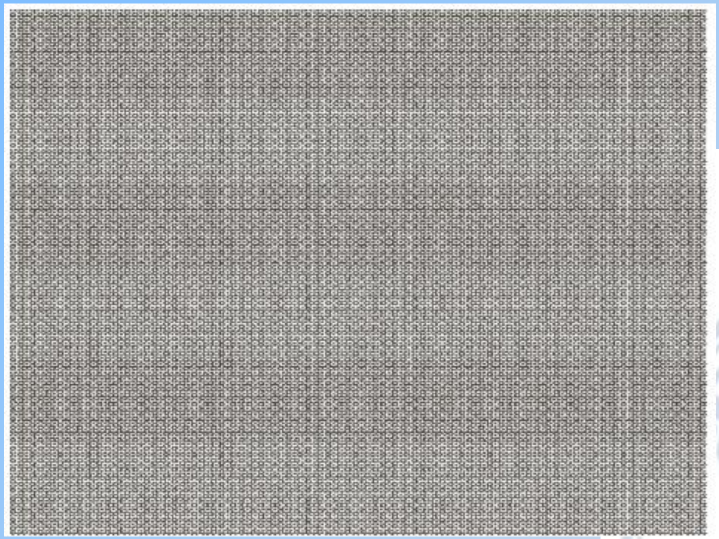


FIGURE 5.17 Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.

Periodic Noise Reduction by Frequency Domain Filtering

- Notch filters
- A notch filter rejects (or passes) frequencies in predefined neighborhoods about a center frequency.
  - Ideal notch reject filter

$$H(u,v)=\begin{cases}0, & D_1(u,v)\le D_0 \text{ or } D_2(u,v)\le D_0\\ 1, & \text{otherwise}\end{cases}$$

where

$$D_1(u,v)=\left[\left(u-\frac{M}{2}-u_0\right)^2+\left(v-\frac{N}{2}-v_0\right)^2\right]^{\frac{1}{2}}$$

$$D_2(u,v)=\left[\left(u-\frac{M}{2}+u_0\right)^2+\left(v-\frac{N}{2}+v_0\right)^2\right]^{\frac{1}{2}}$$

Periodic Noise Reduction by Frequency Domain Filtering

- Notch filters
  - Butterworth notch reject filter

$$H(u,v)=\frac{1}{1+\left[\frac{D_0^2}{D_1(u,v)D_2(u,v)}\right]^n}$$

Periodic Noise Reduction by Frequency Domain Filtering

- Notch filters
  - Gaussian notch reject filter

$$H(u,v) = 1 - e^{-\frac{1}{2}\left[\frac{D_1(u,v)D_2(u,v)}{D_0^2}\right]}$$

Periodic Noise Reduction by Frequency Domain Filtering

- Notch filters

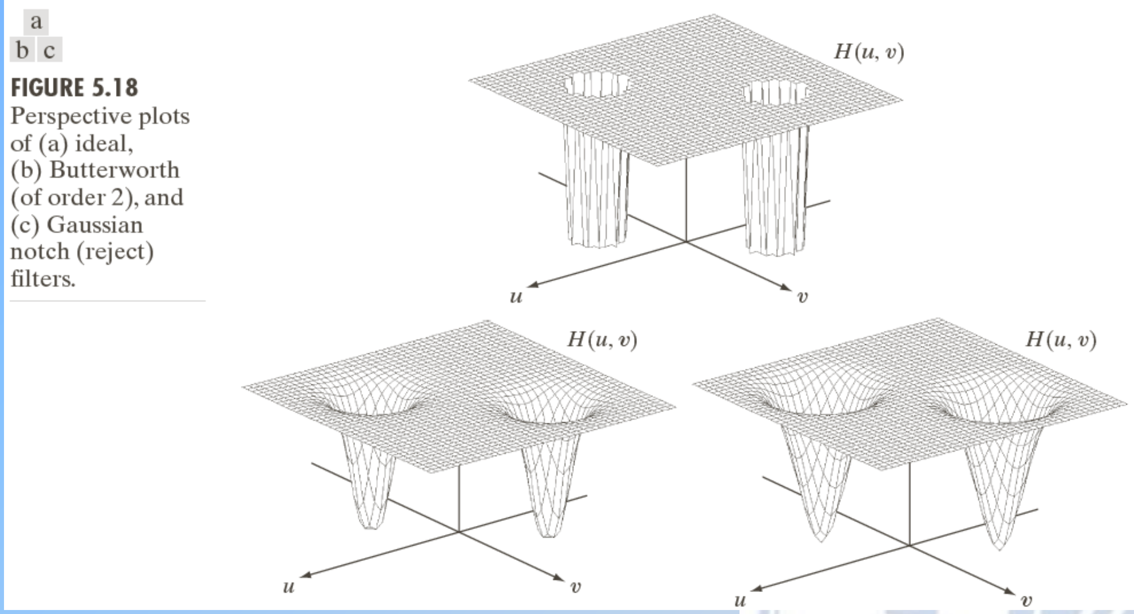


FIGURE 5.18 Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.

Periodic Noise Reduction by Frequency Domain Filtering

- Notch filters
  - Notch pass filters perform exactly the opposite function as the notch reject filters, their transfer functions are given by

$$H_{NP}(u,v) = 1 - H_{NR}(u,v)$$

  - Ideal notch pass filter

$$H(u,v) = \begin{cases} 1, & D_1(u,v) \le D_0 \text{ or } D_2(u,v) \le D_0 \\ 0, & \text{otherwise} \end{cases}$$

Periodic Noise Reduction by Frequency Domain Filtering

- Notch filters
  - Butterworth notch pass filter

$$H(u,v) = 1 - \frac{1}{1 + \left[\frac{D_0^2}{D_1(u,v) D_2(u,v)} \right]^n}$$

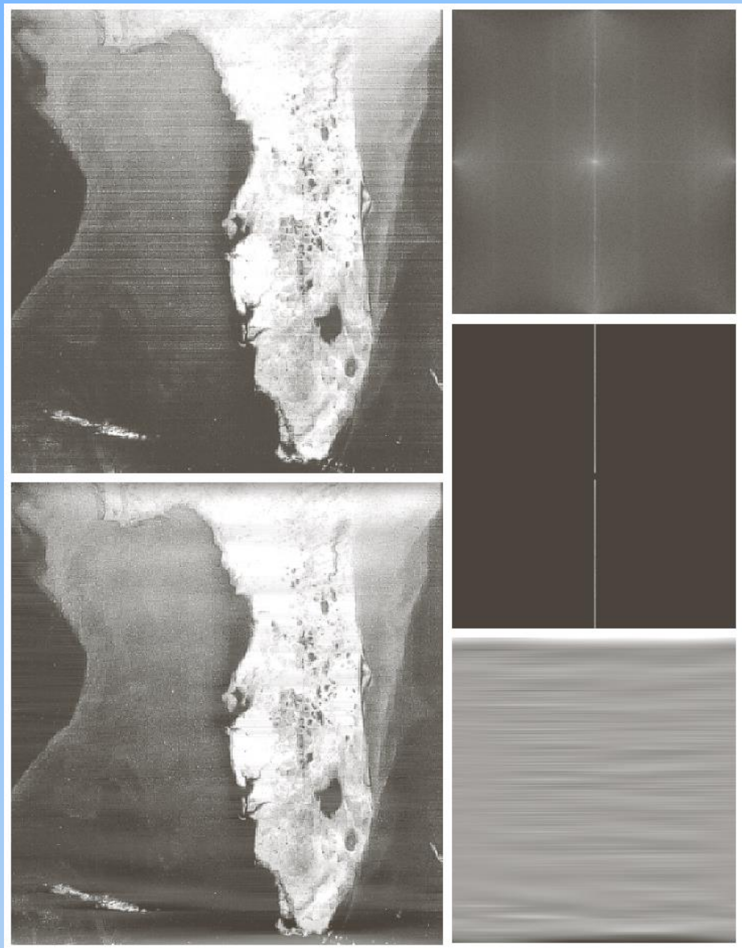
Periodic Noise Reduction by Frequency Domain Filtering

- Notch filters
  - Gaussian notch pass filter

$$H(u,v) = e^{-\frac{1}{2}\left[\frac{D_1(u,v)D_2(u,v)}{D_0^2}\right]}$$

Periodic Noise Reduction by Frequency Domain Filtering

- Notch filters



a b
c
e d

FIGURE 5.19 (a) Satellite image of Florida and the Gulf of Mexico showing horizontal scan lines. (b) Spectrum. (c) Notch pass filter superimposed on (b). (d) Spatial noise pattern. (e) Result of notch reject filtering. (Original image courtesy of NOAA.)

Optimum Notch Filter

The need of an optimum notch filter arises from the fact that clear noise pattern in the Fourier transformed plane are not common.

Consider the image shown below from a spacecraft, the start like bright spots in the Fourier transformed plane (on the right) are not all due to only one type of noises. Instead, it is the combination of several types of noises. In such a case, previous approaches fail.

a b

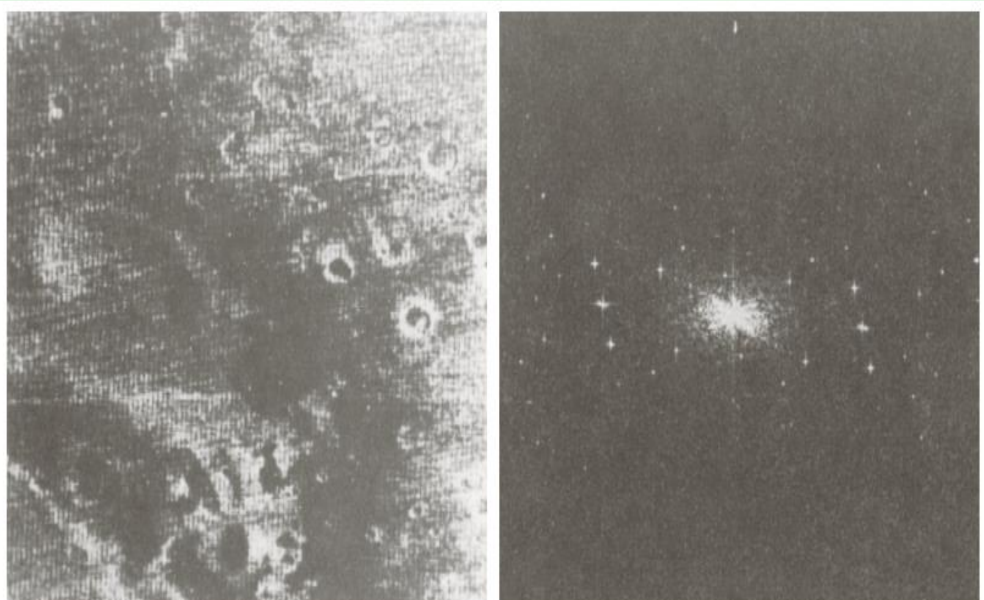


FIGURE 5.20 (a) Image of the Martian terrain taken by *Mariner 6*. (b) Fourier spectrum showing periodic interference. (Courtesy of NASA.)

Let $\eta(x,y)$ be the noise pattern, N(u, v) be its Fourier transform, G(u, v) be the Fourier transform of the noise corrupted image, and a filter H(u, v) is designed to allow only the noise pattern to pass, that is

$$N(u,v) = H(u,v)G(u,v) \tag{5.4.11}$$

Accordingly, the noise pattern $\eta(x,y)$ can be reconstructed from

$$\eta(x,y) = \mathcal{F}^{-1}\{H(u,v)G(u,v)\} \tag{5.4.12}$$

However, in many cases, $\eta(x,y)$ can not be reconstructed exactly. In such a case, the image $\hat{f}(x,y)$ is to be reconstructed from the weighted noise

$$\hat{f}(x,y) = g(x,y) - w(x,y)\eta(x,y) \tag{5.4.13}$$

where $w(x,y)$ is a position dependent weighting function to minimize the local

variance of $\hat{f}(x,y)$, denoted as $\sigma^2(x,y)$, in an neighbor around (x, y) of the size $(2a+1)$x$(2b+1)$

$$\sigma^2(x,y) \triangleq \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{s=-b}^{b} \left[\hat{f}(x+s,y+t) - \overline{\hat{f}}(x,y) \right] \quad (5.4.14)$$

$$\overline{\hat{f}}(x,y) \triangleq \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{s=-b}^{b} \hat{f}(x+s,y+t)$$

$$= \text{the average value of } \hat{f}(s,t) \text{ in the region} \quad (5.4.15)$$

$$s \in [-a+x,\ a+x] \text{ and } t \in [-b+y,\ b+y]$$

Following the derivations given in page 364 of the textbook,it can be shown

$$w(x,y) = \frac{\overline{g(x,y)\eta(x,y)} - \overline{g}(x,y)\overline{\eta}(x,y)}{\overline{\eta^2}(x,y) - \overline{\eta}^2(x,y)}$$

Example

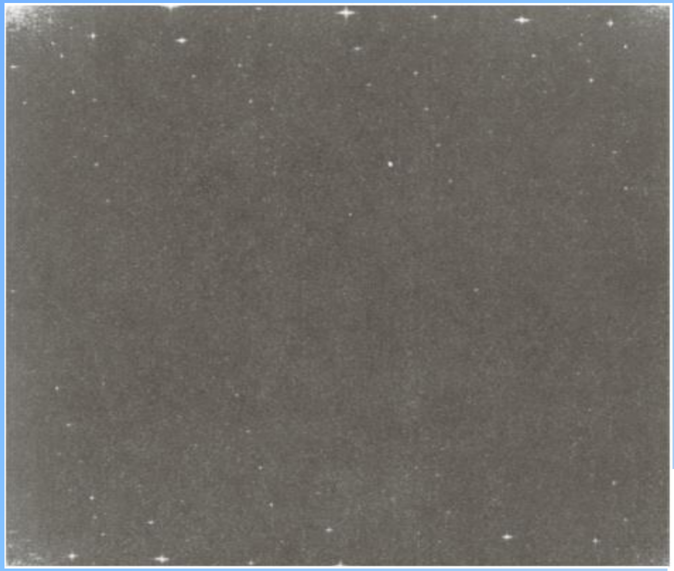
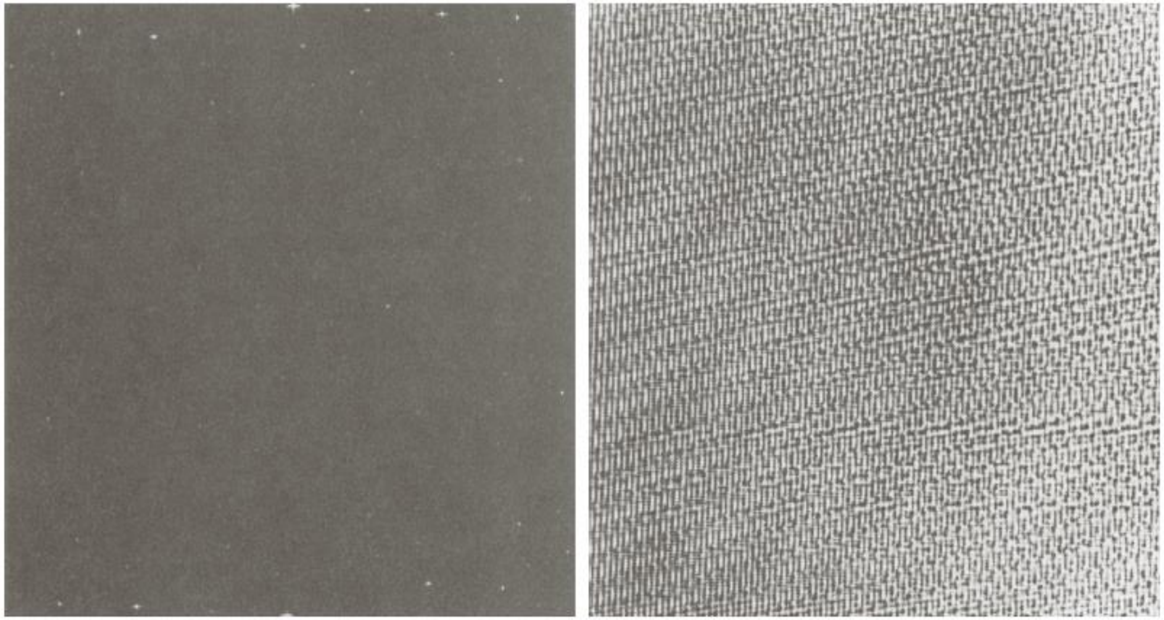


FIGURE 5.21 Fourier spectrum (without shifting) of the image shown in Fig. 5.20(a). (Courtesy of NASA.)

Example



a b

FIGURE 5.22 (a) Fourier spectrum of $N(u, v)$, and (b) corresponding noise interference pattern $\eta(x, y)$. (Courtesy of NASA.)

Example

FIGURE 5.23 Processed image. (Courtesy of NASA.)

Inverse Filtering

- Direct inverse filtering
  - An estimate of the transform of the original image is computed by dividing the transform of the degraded image by the degradation function:

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

  - Plugging G(u, v) = F(u, v) H(u, v) + N(u, v) into the equation yields:

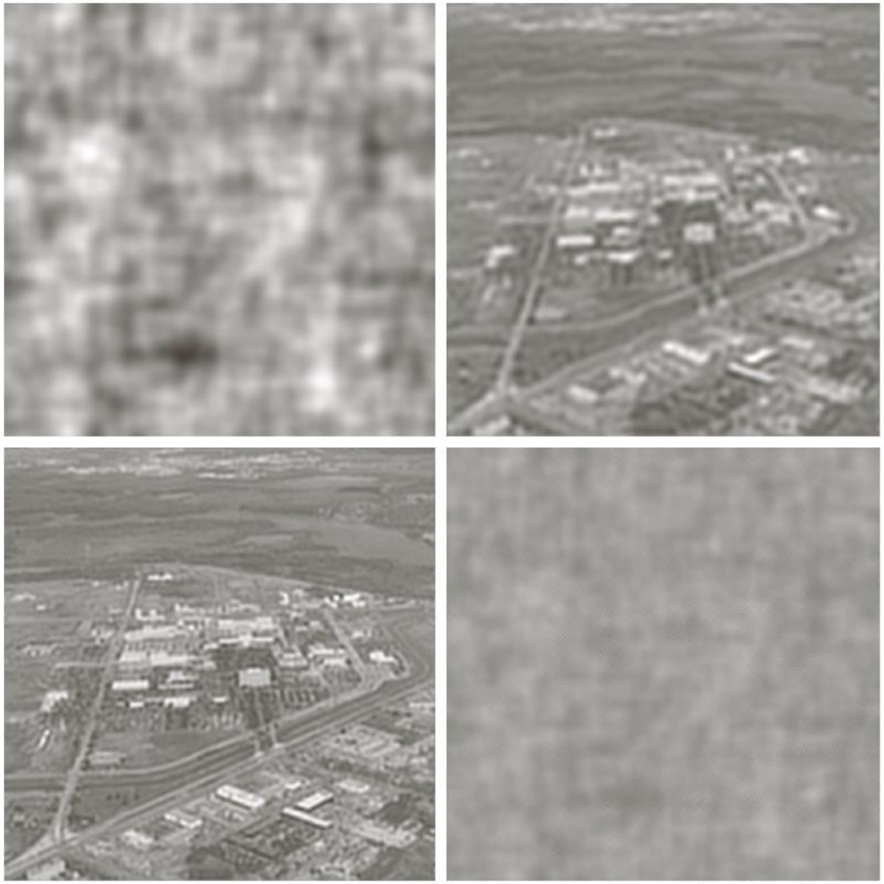
$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

  - Original image can't be recovered even if we know the degradation function!

Inverse Filtering

a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.



Minimum Mean Square Error (Wiener) Filtering

- Wiener filter
  - Both the degradation function and statistical characteristics of noise are incorporated into the restoration process.
  - Consider images and noise as random variables, the objective is to find an estimate $\hat{f}$ of the uncorrupted image f that minimizes the mean square error given by:

$$e^2 = E(f - \hat{f})^2$$

  - Approximation of the above mean square error involving the original and restored images:

$$MSE = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f(x,y) - \hat{f}(x,y)]^2$$

Minimum Mean Square Error (Wiener) Filtering

- Wiener filter
  - Assuming that the noise and the image are uncorrelated; that one or the other has zero mean; and that the intensity levels in the estimate are a linear function of the mean square error measure:

$$
\begin{aligned}
\hat{F}(u,v) &= \frac{H^*(u,v)S_f(u,v)}{S_f(u,v)|H(u,v)|^2 + S_\eta(u,v)}G(u,v) \\
&= \frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}G(u,v) \\
&= \frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}G(u,v)
\end{aligned}
$$

$$S_\eta(u,v) = |N(u,v)|^2 = power\ spectrum\ of\ the\ noise$$

$$S_f(u,v) = |F(u,v)|^2 = power\ spectrum\ of\ the\ undegraded\ image$$

Minimum Mean Square Error (Wiener) Filtering

- Signal-to-noise ratio

$$SNR = \frac{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}|F(u,v)|^2}{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}|N(u,v)|^2}$$

In spatial domain, we can define a signal-to-noise ratio considering the restored image to be "signal" and the difference between this image and the original to be noise:

$$SNR = \frac{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}\hat{f}(x,y)^2}{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}|N(u,v)|^2[f(x,y)-\hat{f}(x,y)]^2}$$

Minimum Mean Square Error (Wiener) Filtering

a b c

FIGURE 5.28 Comparison of inverse and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

Minimum Mean Square Error (Wiener) Filtering

FIGURE 5.29 (a) 8-bit image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)–(f) Same sequence, but with noise variance one order of magnitude less. (g)–(i) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a "curtain" of noise.

Constrained Least Squares(Regularized) Filtering

- The definition of 2-D discrete convolution is

$$h(x,y)*f(x,y)=\frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}f(m,n)h(x-m,y-n)$$

- In vector-matrix form, as

$$g=Hf+\eta$$

Constrained Least Squares(Regularized) Filtering

- Matlab program(download from the course's home page)
  - Restoration.m
  - Est_noise.m

Constrained Least Squares(Regularized) Filtering

- To be meaningful, the restoration must be constrained by the parameters of the problem at hand. Thus, what is desired is to find the minimum of a criterion function, C, defined as

$$C = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[\nabla^2 f(x, y) \right]^2$$

subject to the constraint

$$\left\| g - H\hat{f} \right\|^2 = \left\| \eta \right\|^2$$

Constrained Least Squares(Regularized) Filtering

- The frequency domain solution to this optimization problem is given by the expression

$$\hat{F}(u,v)=\left[\frac{H^*(u,v)}{|H(u,v)|^2+\gamma|P(u,v)|^2}\right]G(u,v)$$

- P(u, v)is the Fourier transform of the function

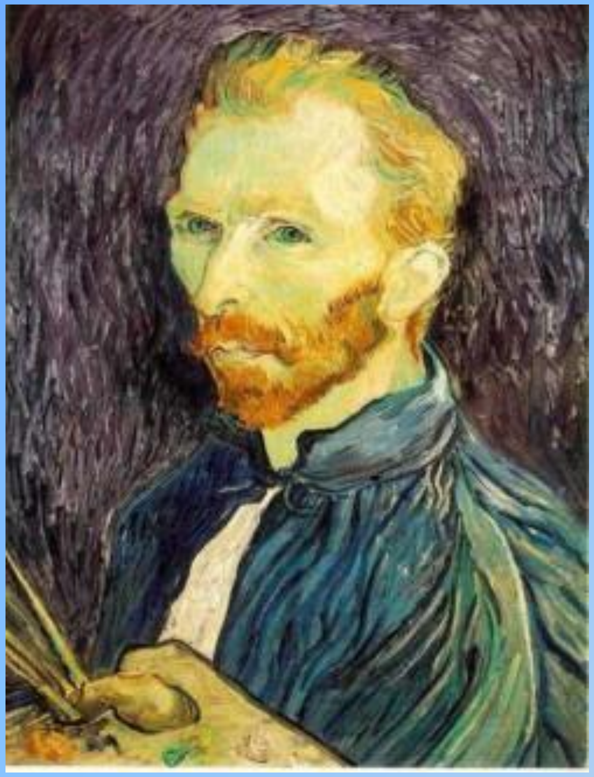
$$p(x,y)=\begin{bmatrix}0 & 1 & 0\\ 1 & -4 & 1\\ 0 & 1 & 0\end{bmatrix}$$

Constrained Least Squares(Regularized) Filtering

- Matlab program
  - Restoration_ls.m

- There are also Wavelet-based Image Restoration, Blind Deconvolution, which are outside the scope of the present discussion.

Image Restoration



filter the image, *then* subsample

Image Restoration

The degradation process:
blurring & down sample

Image Super-Resolution Reconstruction

In many real-world application scenarios such as military transmission, medical science, and astronomy, high-resolution images or videos are often required while only low-quality images or videos are available due to the limited bandwidth or storage. Therefore, the problem to reconstruct high-resolution versions from the quality degraded sources has attracted many attentions.

Image Super-Resolution Reconstruction

Super-resolution are techniques that enhance the resolution of an imaging system. There are mainly two ways to generate a super-resolution image: from a single low-resolution image, and from multiple low-resolution images of the same scene.

Multiple Low-Resolution Images SR

- One possible method: Projection on Convex Sets (POCS), or Convex Projection
- The original f is a vector known, a priori, to belong to a linear subspace S of a parent Hilbert space H, but all that is available to the observer is the orthogonal projection g of f onto another linear subspace J (also in H).
- Given the partial data g, 1) find necessary and sufficient conditions for the unique determination of f from g, and 2) find necessary and sufficient conditions for the stable linear reconstruction of f from g in the face of noise.
- The answers turn out to be quite simple.

Multiple Low-Resolution Images SR

- 1) f is uniquely determined by g iff S and the orthogonal complement of J only have the zero vector in common.
- 2) The reconstruction problem is stable iff the angle between g and the orthogonal complement of f is greater than zero.
- 3) In the absence of noise, there exists in both cases 1) and 2) an effective recursive algorithm for the recovery of f employing only the operations of projection onto S and projection onto the orthogonal complement of J.

Multiple Low-Resolution Images SR

- The conceptual basis for the algorithm is somewhat similar to that for the linear case. The original f is known, a priori, to belong to the intersection C_o of m well-defined closed convex sets C_1, C_2, ..., C_m

$$f \in C_0 = \bigcap_{i=1}^{m} C_i$$

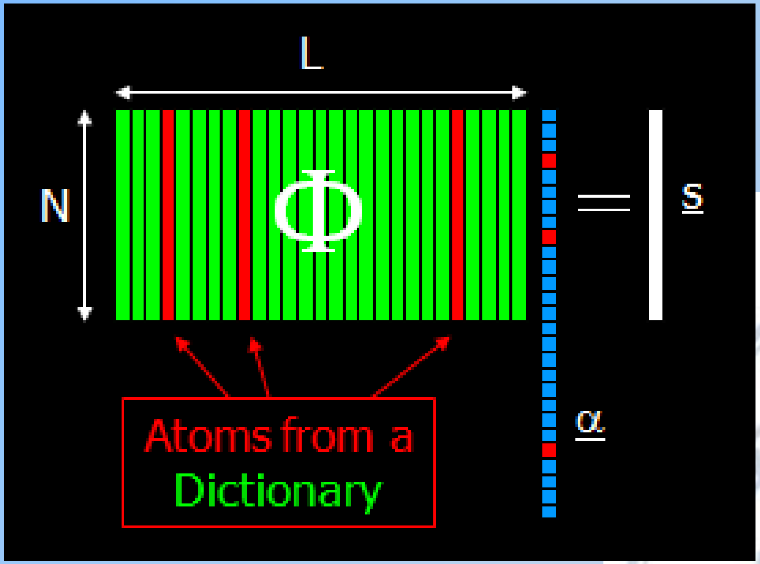
- Given only the (nonlinear) projection operators P_i onto the individual C_i's, i = 1 ,..., m, restore f, preferably by an iterative scheme. Thus, the realization of the Pi's is the major synthesis problem in an arbitrary Hilbert space setting.

Multiple Low-Resolution Images SR

- Program: multiple low-resolution images of the same sense super-resolution reconstruction
  - ./POCS/pocs.m
  - 8 images, 10 iteration
- Learn more about POCS
  - Image Restoration by the Method of Convex Projections: Part 1-Theory, D. C. YOULA AND H. WEBB
  - Image Restoration by the Method of Convex Projections: Part 2-Applications and Numerical Results, M. I. SEZAN AND H. STARK

The Linear Transforms

- Special interest - linear transforms (inverse) $\underline{s} = \Phi \underline{\alpha}$



- In square linear transforms, Φ is an N-by-N & non-singular.

Matching Pursuit

- Given d unitary matrices $\{\Phi_k, 1 \le k \le d\}$, define a dictionary $\Phi = [\Phi_1, \Phi_2, \dots \Phi_d]$ [Mallat & Zhang (1993)].
- Combined representation per a signal $\underline{s}$ by

$$\underline{s} = \Phi \underline{\alpha}$$

- Non-unique solution $\underline{\alpha}$ - Solve for maximal sparsity

$$P_0: \quad \underset{\underline{\alpha}}{\text{Min}} \ \|\underline{\alpha}\|_0 \quad \text{s.t.} \quad \underline{s} = \Phi \underline{\alpha}$$

- Hard to solve – a sub-optimal greedy sequential solver: "Matching Pursuit algorithm" .

Matching Pursuit

- **Matching pursuit** is a type of numerical technique which involves finding the "best matching" projections of multidimensional data onto an over-complete dictionary *D*. The basic idea is to represent a signal *f* from Hilbert space *H* as a weighted sum of functions g_{γ_n} (called atoms) taken from *D*:

$$f(t) = \sum_{n=0}^{+\infty} a_n g_{\gamma_n}(t)$$

Matching Pursuit

- Searching over an extremely large dictionary for the best matches is computationally unacceptable for practical applications. In 1993 Mallat and Zhang proposed a greedy solution that is known from that time as Matching Pursuit. The algorithm iteratively generates for any signal *f* and any dictionary *D* a sorted list of indexes and scalars which are sub-optimal solution to the problem of sparse signal representation:

Algorithm Matching Pursuit

Input: Signal: $f(t)$.

Output: List of coefficients: (a_n, g_{γ_n}).

Initialization:

$Rf_1 \leftarrow f(t)$;

$n \leftarrow 1$;

Repeat

find $g_{\gamma_n} \in D$ with maximum inner product $\langle Rf_n, g_{\gamma_n} \rangle$;

$a_n \leftarrow \langle Rf_n, g_{\gamma_n} \rangle$;

$Rf_{n+1} \leftarrow Rf_n - a_n g_{\gamma_n}$;

$n \leftarrow n+1$;

Until stop condition (for example: $||Rf_n|| < threshold$)

Basis Pursuit

- **Facing the same problem, and the same optimization task** [Chen, Donoho, Saunders (1995)]

$$P_0: \quad \underset{\underline{\alpha}}{Min} \quad \|\underline{\alpha}\|_0 \quad s.t. \quad \underline{s} = \Phi\underline{\alpha}$$

Hard to solve – replace the l_0 norm by an : “Basis Pursuit algorithm”

$$\mathrm{P}_1: \quad \underset{\underline{\alpha}}{\mathrm{Min}} \quad \|\underline{\alpha}\|_1 \quad \mathrm{s.t.} \quad \underline{\mathrm{s}} = \Phi\underline{\alpha}$$

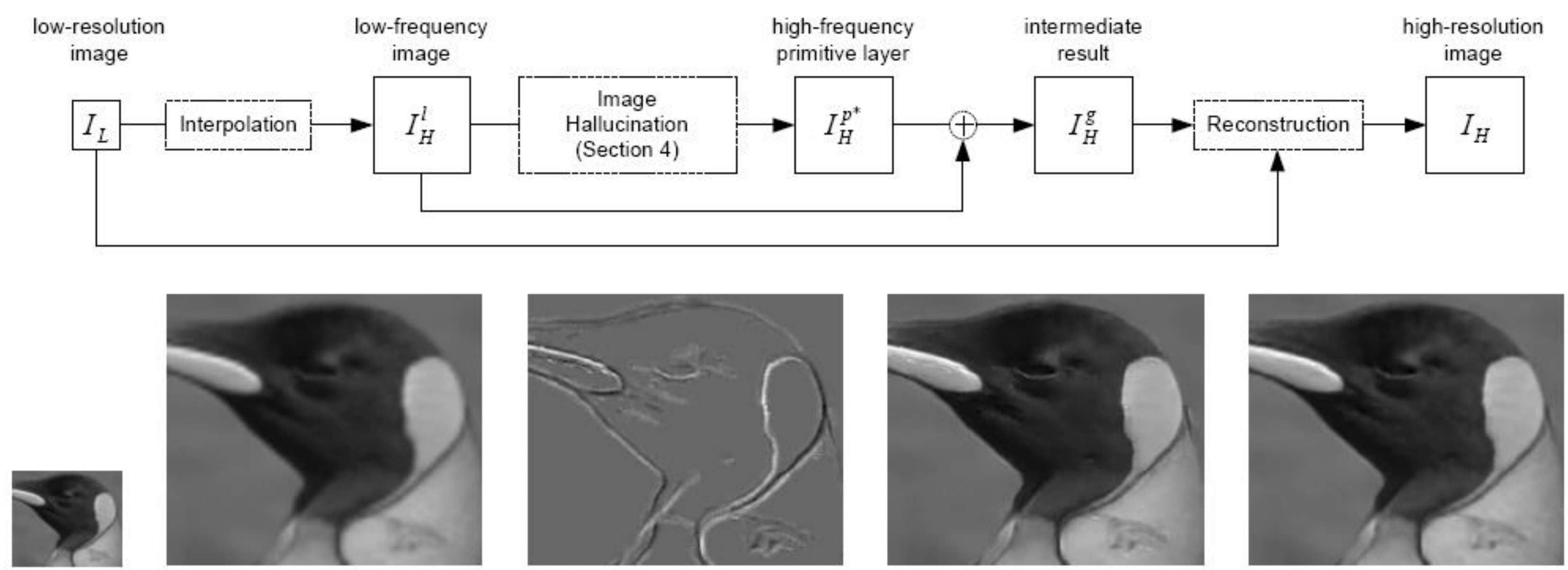
- Interesting observation: In many cases it successfully finds the sparsest representation. [Optimally sparse representation in general (no orthogonal) dictionaries via l_1 minimization]

Single Image Super-Resolution Reconstruction

- Methods of single image super-resolution can be broadly classified into three families: interpolation-based, reconstruction-based and learning-based.
  - Interpolation-based methods are based on the assumption of the strong correlations between adjacent pixels and most of them are efficient to be conducted.
  - Reconstruction-based methods introduce the prior knowledge as reconstruction constraints when regularizing the super-resolution image.
  - Learning-based methods infer the lost high-frequency information using a learned co-occurrence prior knowledge.
- Currently, the learning-based methods have shown their promises in super-resolution.

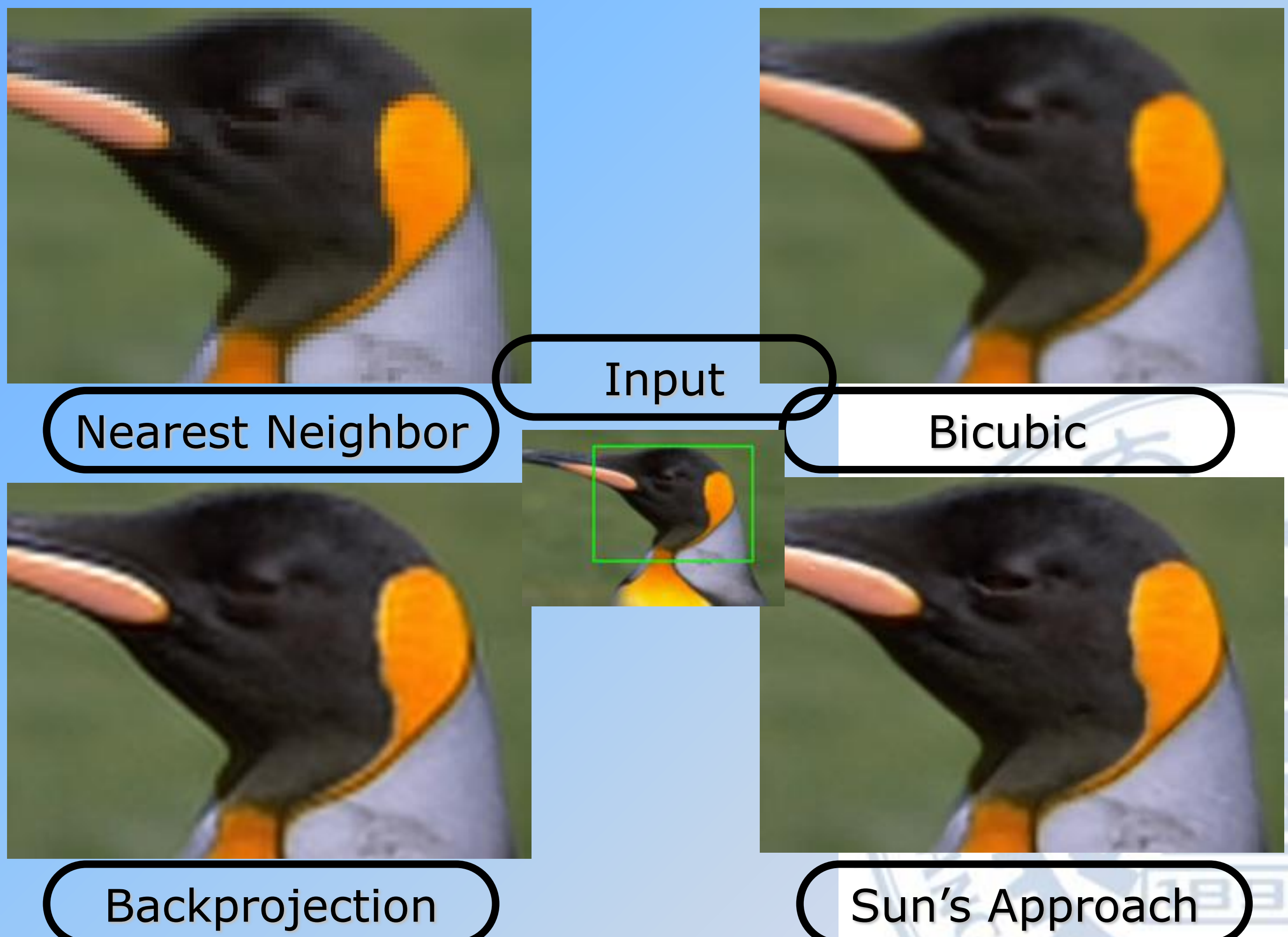
Single Image Super-Resolution Reconstruction

- Jian Sun et al. propose a Bayesian approach to image hallucination. Given a generic low resolution image, they hallucinate a high resolution image using a set of training images.



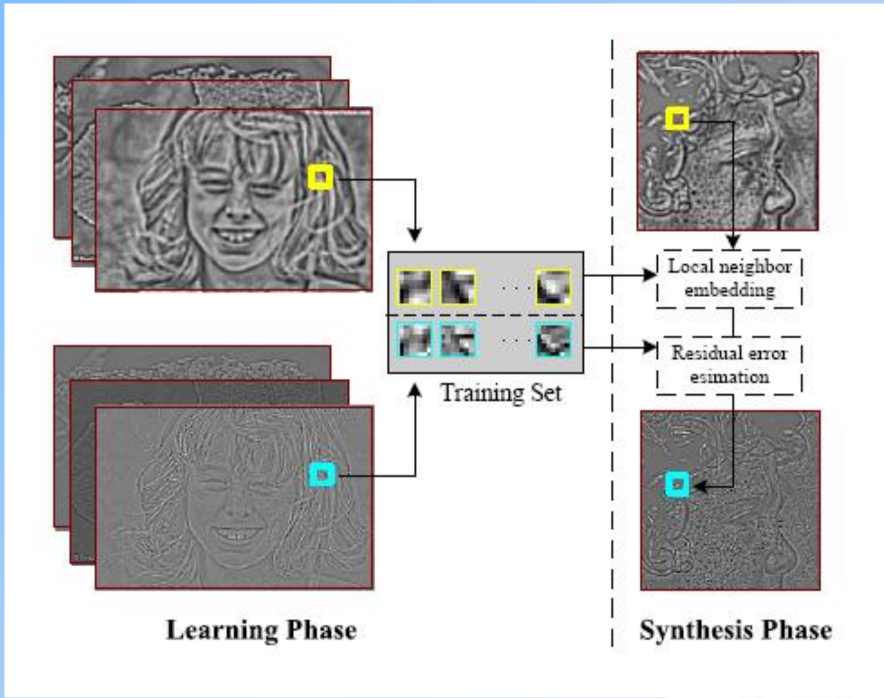
(a) I_L (b) I_H^l (c) I_H^{p*} (d) I_H^g (e) I_H

Image Hallucination with Primal Sketch Priors, Jian Sun



Single Image Super-Resolution Reconstruction

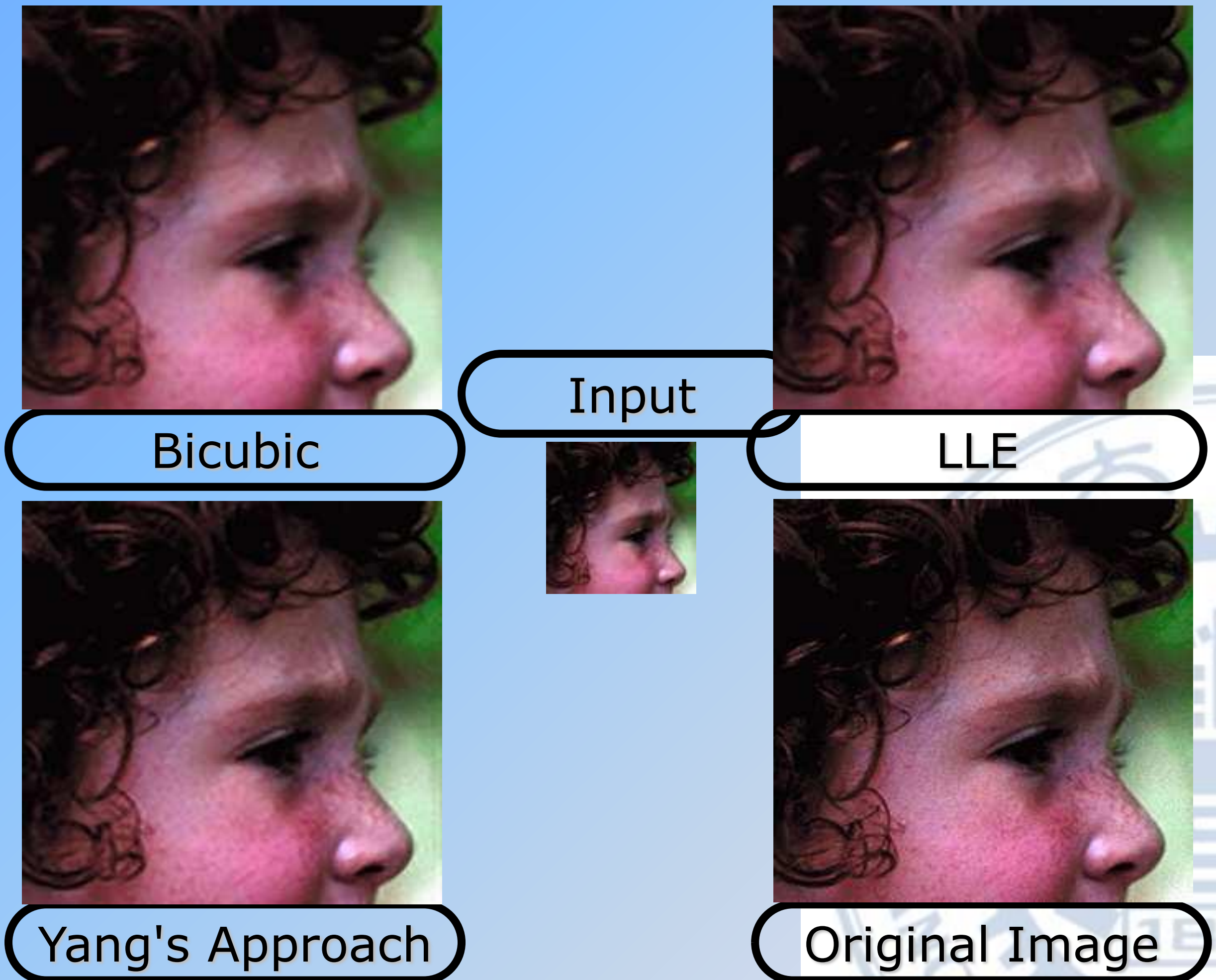
- Inspired by recent progress in manifold learning research, Wei Fan et al. take the assumption that small image patches in the low-resolution and high-resolution images form manifolds with similar local geometry in the corresponding image feature spaces. [Image Hallucination Using Neighbor Embedding over Visual Primitive Manifolds]



Single Image Super-Resolution Reconstruction

- Jianchao Yang presents a new approach to single-image super-resolution, based on sparse signal representation. [Image Super-Resolution via Sparse Representation, IEEE transactions on image processing] [Image Super-Resolution as Sparse Representation of Raw Image Patches]
- By jointly training two dictionaries for the low resolution and high resolution image patches, enforce the similarity of sparse representations between the low resolution and high resolution image patch pair with respect to their own dictionaries.
- They seek a sparse representation for each patch of the low-resolution input, and then use the coefficients of this representation to generate the high-resolution output.

LLE Method & Yang's Work



Proposed Method

Original Image

Low-resolution

Bicubic Interpolation

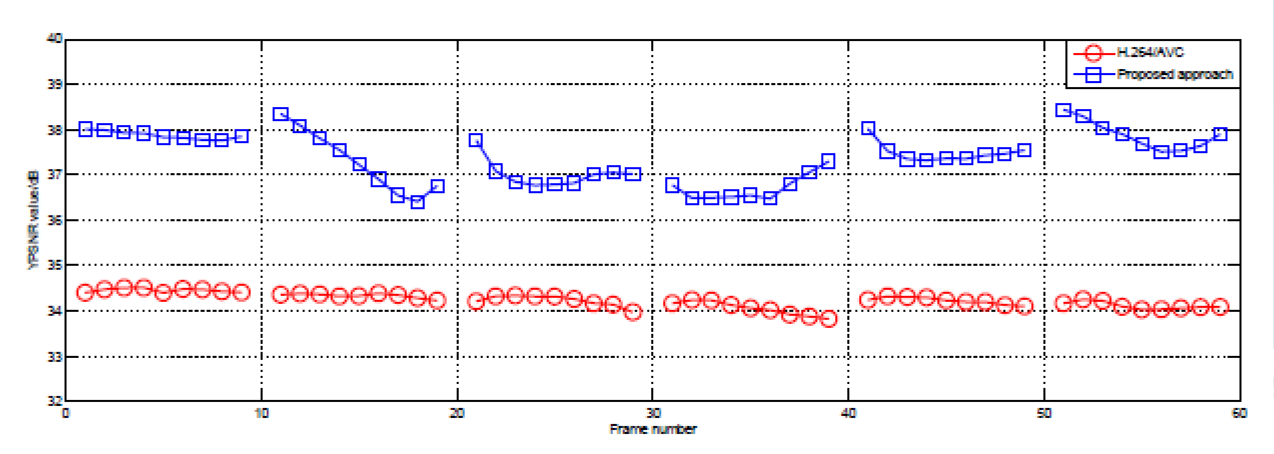
Proposed Approach

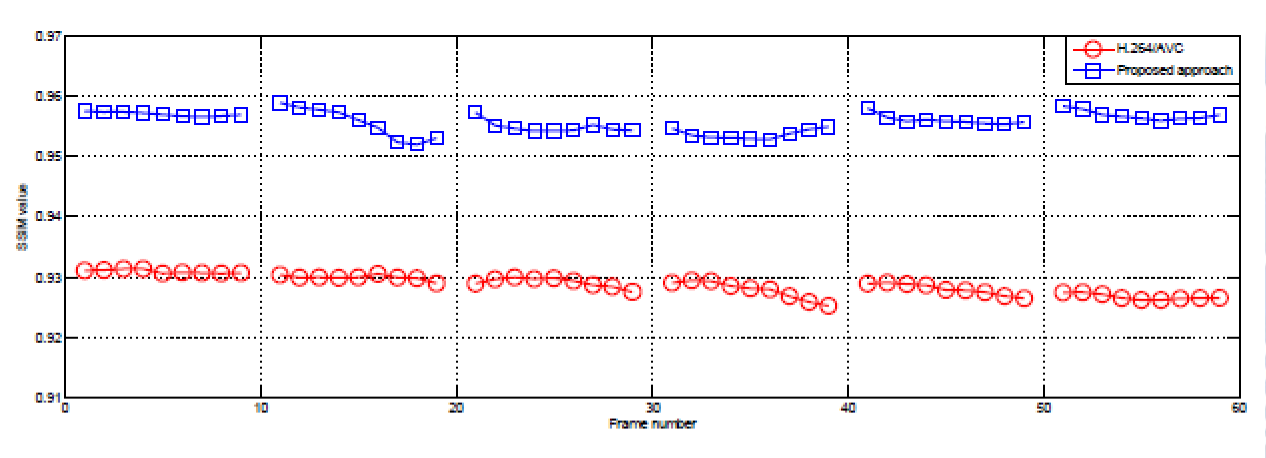
SR in Video Compression

H.264

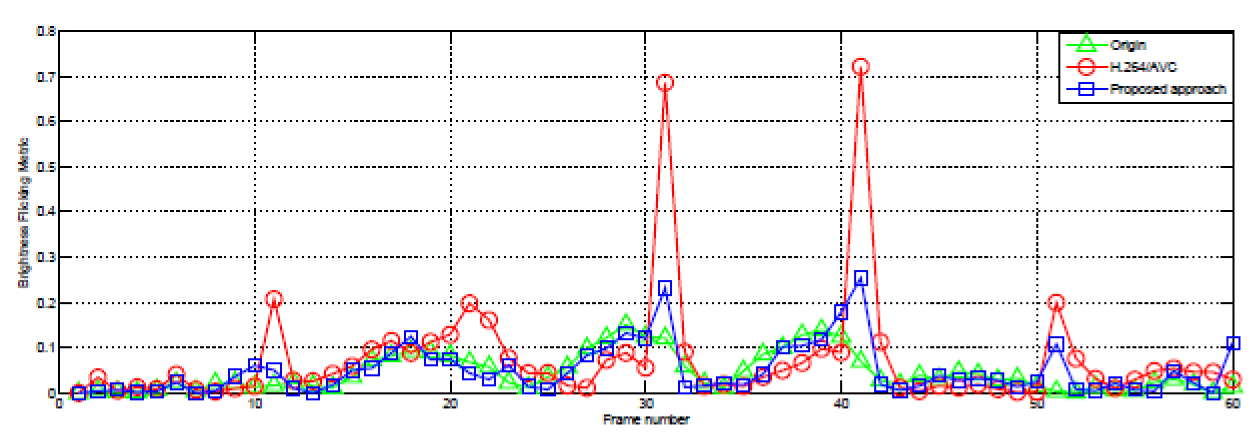
With SR

SR in Video Compression





SR in Video Compression



Requirements of Project Two now posted!

Thank You!